The apprpximate method and its application for the AKNS–type linear scattering problem by descritizing the initial wave packet

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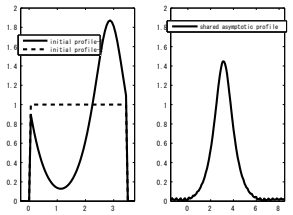
Abstract:

The Zakharov–Shabat eigenvalue problem for the AKNS form[1]

$$\begin{pmatrix} \Psi_{1x} \\ \Psi_{2x} \end{pmatrix} = \begin{pmatrix} \xi & q(x,t) \\ r(x,t) & -\xi \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \tag{1}$$

is a universal framework to describe many soliton equations which appear in various fields of physics. These equations are integrable, however, the analysis of their initial value problems beyond formal solutions is hopeless. For general initial pulses that deviate from pure soliton solutions, it is difficult to apply the Hirota's method or directly solve the Gelfand–Levitan–Marchenko equation[2]. However, it is at least desirable to obtain some asymptotic ($t \to \infty$) information of the solution. Recently, we have found an asymptotic method for this problem, in which we divide the initial wave packet into many small intervals and consider the transfer matrices connecting each of them. We can extract the asymptotic information of the wave packet starting from arbitrary initial conditions. We have also applied our method to the nonlinear Schrödinger equation and found the non monotonic increase of the number of asymptotic solitons as a function of initial amplitude for double box type initial conditions. Moreover, using the Darboux transformation, we analytically show the class of initial profile in which the asymptotic solitons share the same amplitude and width. The possibility of classifying the set of initial conditions into equivalence classes represented by each asymptotic solitons is addressed.



References:

1. V.E. Zakharov and A.B. Shabat: Sov.Phys. JETP **34** (1972) 62.
2. M. J. Ablowitz, H. Segur, "Solitons and the Inverse Scattering Transform", SIAM (1985).